

Indian Statistical Institute
B.Math. (Hons.) II Year
Second Semester 2006-07
Mid-Sem Examination
Algebra IV

Time: 3 hrs

Date:07-03-07

Instructor: M Thakur

All questions carry equal marks. Attempt all the questions.

1. Prove that the polynomial $P(X) = X^3 - 3X - 1$ is irreducible over \mathbb{Q} , the field of rational numbers.
2. Compute the degree $[\mathbb{Q}(\sqrt[3]{2} + \sqrt{5}) : \mathbb{Q}]$.
3. Prove that $8X^3 - 6X - 1$ is irreducible over \mathbb{Q} .
4. Let $\text{char } k = p > 0$ and $f(X) \in k[X]$ be such that $f'(X) = 0$. Prove that \exists a polynomial $g(X) \in k[X]$ such that

$$f(X) = g(X^p).$$

5. Let H and K be subgroups of G , $|H|^2 > |G|$, $|K|^2 > |G|$. Show that $H \cap K \neq \{1\}$. Here $||$ denote the order.
6. Let $n \geq 3$. Prove that the cycle $(1\ 2\ 3)$ is not the cube of any element in S_n .
7. In S_n , prove that conjugate of a cycle of length r is a cycle of length r .
8. Determine the number of conjugacy class in S_4 .
9. Find the number of non-isomorphic abelian groups of order 81.
10. Let $K = \mathbb{Q}(\alpha_1, \dots, \alpha_n)$ where $\alpha_i^2 \in \mathbb{Q}$ for $1 \leq i \leq n$. Prove that $\sqrt[3]{2} \notin K$.